A Micromechanical Explanation of Transformation Induced Plasticity (TRIP) Applied to Austenite-Martensite Transformation

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Introduction

TRIP is the remaining deformation behaviour observed when a specimen transforms under an applied stress even for stresses lower than the yield stress of the material. Denis et al. (1), gave in a review paper the following two actual interpretations:

- Deformation due to a plastic yielding around the transforming particles due to the applied stress taking into account only the volume change.
- Deformation due to a specific orientation of the product phase by the applied stress .

The first interpretation goes back to the famous paper by Greenwood et al. (2), and assumes the transformed regions to be "hard" islands in a "weak" matrix. Although the model has some shortcomings it was successfully applied to diffusional transformations (e.g. austenite-pearlite transformation). Abrassart and later Leblond, see discussion (1), used this concept also for a martensitic transformation only considering the volume change.
The second interpretation goes back to Magee, (3). Modelling is carried out on the microscopical scale of the martensite plate. If one assumes a simple crystal with one martensite variant the "free" transformation strain tensor $\underline{\varepsilon}_o'$ can be written with respect to the habit plane (common crystallographic plane between the both phases, coordinate x', y' in the habit plane, z' orthogonal to habit plane) as

$$\underline{\varepsilon}_c' = \begin{bmatrix} 0 & 0 & \gamma/2 \\ 0 & 0 & 0 \\ \gamma/2 & 0 & \delta \end{bmatrix} , \qquad (1)$$

γ is the transformation shear angle and may have a typical value of 0.20. δ is the (anisotropic) volume change with a typical value of 0.04 . Connecting the local coordinate system x', y', z' with the global coordinate system x, y, z by the three Eulerian angles ψ , φ , Θ and a corresponding transformation matrix $\underline{\Omega}$ the "free" transformation strain components of $\underline{\varepsilon}_o$ in the global system are as follows

$$\varepsilon_{o,x} = \gamma/2(\sin2\psi\sin\Theta\cos\varphi - \sin^2\psi\sin2\Theta\sin\varphi) + \delta(\sin^2\psi\sin^2\Theta) , \qquad (2a)$$
$$\varepsilon_{o,y} = -\gamma/2(\sin2\psi\sin\Theta\cos\varphi + \cos^2\psi\sin2\Theta\sin\varphi) + \delta(\cos^2\psi\sin^2\Theta) , \qquad (2b)$$
$$\varepsilon_{o,z} = \gamma/2(\sin2\Theta\sin\varphi) + \delta/2(1+\cos2\Theta) , \qquad (2c)$$
$$\gamma_{o,xz} = \gamma(\cos\psi\cos\Theta\cos\varphi - \sin\psi\cos2\Theta\sin\varphi) + \delta\sin\psi\sin2\Theta , \qquad (2d)$$
$$\gamma_{o,yz} = \gamma(\sin\psi\cos\Theta\cos\varphi + \cos\psi\cos2\Theta\sin\varphi) - \delta\cos\psi\sin2\Theta , \qquad (2e)$$
$$\gamma_{o,xy} = \gamma(-\cos2\psi\sin\Theta\cos\varphi + 1/2\sin2\psi\sin2\Theta\sin\varphi) - \delta/2\sin2\psi\sin^2\Theta , \qquad (2f)$$

Details can be taken from Mitter's book, (4), chapter 2.2.1.1.8. The role of the stress is now only to select the martensite variant. If a specimen is loaded only in the z-direction the resulting transformation strain is $\varepsilon_{c,z}$. Magee, (3), argued now that $\varphi = \pm \pi/2$ is the most probable direction with respect to the "principle of the least constraint". Then it follows with (2c) for the maximum value of $\varepsilon_{c,z}$

$$\varepsilon_{c,z,max} = 1/2\ (\delta \pm \sqrt{\gamma^2 + \delta^2})\ \text{with}\ \Theta = 1/2\ \text{atan}\,\gamma/\delta\ . \qquad (3)$$

This leads with $\gamma = 0.2$, $\delta = 0.04$ to a reversible longitudinal remaining strain of 0.122 independent of the stress level. Magee, (3), suggested a weighted transformation strain as

$$\varepsilon_{z,t} = \int_{\Theta=0}^{\pi} \varepsilon_{c,z}\ (\Theta, \varphi = \pi/2)\ g(\Theta)\ d\Theta\ /\ \int_{\Theta=0}^{\pi} g(\Theta)\ d(\Theta)\ ) \qquad (4)$$

$g(\Theta)$ is a specific weighting function, e.g. described in (6). Gautier et al. discussed this "stress"-free transformation strain recently in (5) e.g. for Cu-Zn-Al single crystal.

In reality both deformation mechanisms - plastification and selection of martensite variants - occur. The "free" transformation strains, (1), (2) have to be accommodated. Depending on these transformation strains internal stresses are generated, and the material will be submitted to the external applied stress and the internal stress field. The "free" transformation strains can be accommodated elastically, elastoplastically or by formation of selfaccommodating plates (concerning the transformation shear strain). The way of accommodation depends on the magnitude of the transformation strains and on the applied stress level.

The "microregion" concept

In the following TRIP is investigated under a constant external load stress field $\underline{\sigma}_l$ applied before transformation ("creep"-test). Now the specimen is assumed to be divided into "microregions". Different possibillities exist as discussed by Mitter, (4), chapter 2.2.1.1.8, Fischer (6) and Pichler (7). A simple proposal, following Fischer, (6), is to take as a microregion one specific martensite plate, see Fig.1

If the "microregion 1" is not restricted by its neighbouring microregions or the remaining parent phase its final deformation would be defined by the "free" transformation strain tensor $\underline{\varepsilon}_c'$, see (1). A reduced shear angle due to self-accommodation can also be considered, see "microregion 2". Principally the deformation behaviour of an isolated microregion is described by $\underline{\underline{T}}_c'$ see (1) with $\overline{\gamma}$, $\overline{\delta}$ as the final deformations. A lot of microregions with different orientations exists in a "mesodomain" as a periodically repeatable part of a specimen under a homogenous load stress field $\underline{\sigma}_l$. A specific microregion is more or less restricted by the neighbouring microregions and parent phase "islands". Following Taylor's "Full Constraint Theory" (as recently discussed by Tiem et al, (8)) now the same global strain tensor $\underline{\varepsilon}_M$ in the sense of a "full" accommodation is assumed in the microregions. Precise measurements on steel cylinders with a transformation fraction $\xi \sim 1$ justify this assumption.

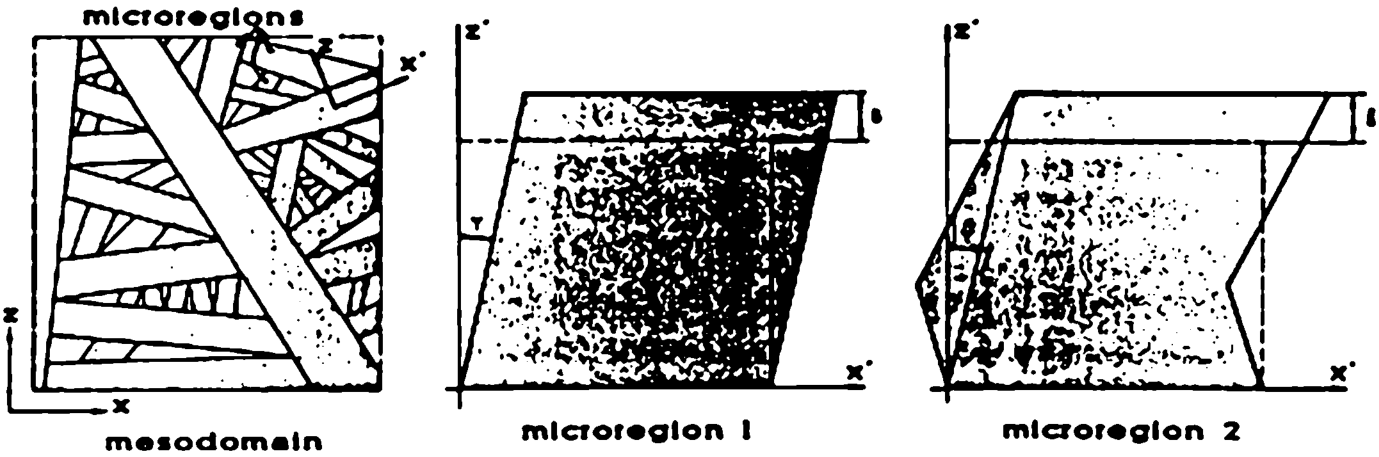


Fig.1: Definition of a microregion

Loaded cylinders keep nearly perfectly their axial symmetric shape during transformation. Also in the remaining parent phase "islands" the same average global strain tensor $\underline{\varepsilon}_o$ is assumed. A high density of dislocations can often be found in the martensitic regions. The neighbouring parent phase is then at least partially plastified. Therefore, the microregions are considered to be plastified. Unless otherwise specified the elastic strain components are assumed to be small compared to $\underline{\varepsilon}_N$, $\underline{\varepsilon}_o$ and are ignored in this concept. In reality a residual stress state exists compensating the actually existing strain differences from microregion to microregion and/or parent phase. The corresponding stored strain energy may play an important role in the transformation kinetics. It should be mentioned that Patoor et al (9) studied recently the interaction of different martensite lenses under an elastic behaviour in a selfconsistent scheme. But a plastification of the different phases is not treated at present. For simplicity an isothermal transformation process with a very rapid cooling to a temperature $\Theta_o < M_s$ is assumed. During the displacive transformation the time dependence of $\underline{\varepsilon}_C$, (1) and $\underline{\varepsilon}_N$ are taken to be an explicit one and the same,

$$\dot{\underline{\varepsilon}}_C = \dot{f}(t)\,\overline{\underline{\varepsilon}}_C \;,\quad \dot{\underline{\varepsilon}}_N = \dot{f}(t)\,\overline{\underline{\varepsilon}}_N \tag{5}$$

With the stress tensor $\underline{\sigma}$, the deviator $\underline{S} = \underline{\sigma} - p\underline{1}$, $p = (\sigma_x + \sigma_y + \sigma_z)/3$, in a microregion standard plasticity results in

$$\dot{\underline{\varepsilon}}_p = \dot{\underline{\varepsilon}}_N - \dot{\underline{\varepsilon}}_C = \dot{f}(t)\,(\overline{\underline{\varepsilon}}_N - \overline{\underline{\varepsilon}}_C) = \lambda\,\underline{S} \;,\quad \underline{S}:\underline{S} = 2/3\,\sigma_f^2 \;. \tag{6}$$

Using the yield condition the following relation between the components of the microregion stress tensor, S_{ij} , and the unknown components of $\overline{\underline{\varepsilon}}_N$, $\overline{\varepsilon}_{N.ij}$ can be formulated

$$S_{ij} = \sigma_f \cdot \frac{(\overline{\varepsilon}_{N.ij} - \overline{\varepsilon}_{C.ij})}{\left\{ 3/2 \left[\overline{\underline{\varepsilon}}_N : (\overline{\underline{\varepsilon}}_N - 2\underline{\varepsilon}_C) + \overline{\gamma}^2/2 + \overline{\delta}^2 \right] \right\}^{1/2}} \;. \tag{7}$$

Since the average stress tensor in a mesodomain is the load stress tensor $\underline{\sigma}_l$ the following relation between the average stress tensor in the "old" regions (volume fraction $1-\xi$), $\langle\underline{\sigma}\rangle_O$, and $\underline{\sigma}_l$ can be written,

$$(1-\xi)\,\langle\sigma_{ij}\rangle_O \;+\; \xi\,\langle\sigma_{ij}\rangle_N \;=\; \sigma_{l,ij}\ , \qquad \text{(8a)}$$

$$(1-\xi)\,\langle S_{ij}\rangle_O \;+\; \xi\,\langle S_{ij}\rangle_N \;=\; S_{l,ij}\ , \qquad \text{(8b)}$$

respectively for the deviatoric components. This relation can also be proved for a piecewise differentiable stress distribution , see (6). The average process may include different types of weighting functions $g(\theta,\psi,\varphi)$, see also (4) and is performed over the Eulerian angles,

$$\langle u\rangle = \int_{\theta=0}^{\pi}\int_{\psi=0}^{2\pi}\int_{\varphi=0}^{2\pi} g(\theta,\phi,\varphi)\,u(\theta,\phi,\varphi)\,\sin\theta\,d\theta\,d\psi\,d\varphi \Big/ \int_\theta\int_\phi\int_\varphi g\sin\theta\,d\theta\,d\phi\,d\varphi\ . \qquad (9)$$

If σ_f and $\langle\sigma_{ij}\rangle_O$ are known, insertion of (7) into (8b) delivers 6 nonlinear algebraic equations for the unknown components of $\underline{\bar{\tau}}_N$, see details in (6), (7). If some components $\bar{\tau}_{N,ij}$ have to be 0, the corresponding equation must be fulfilled in the sense of equilibrum equations. The solution of the equations must be performed numerically.

Examples

For simplicity a fully transformed specimen, $\xi = 1$, and a constant σ_f are assumed. It should be mentioned that σ_f varies from the parent phase yield stress to the product phase yield stress, see discussion (7), chapter 4. Unless otherwise defined, $\bar{\gamma} = 0.2$, $\bar{\delta} = 0.04$.

Specimen under an unaxial loadstress $\sigma_z = \sigma$

$\underline{\bar{\tau}}_N$ has the following components different from 0:

$$\bar{\tau}_{N,x} = \bar{\tau}_{N,y} = x\ ,\quad \bar{\tau}_{N,z} = \varepsilon,\ \varepsilon = 2x + \bar{\delta}\ .$$

Due to Magee's arguments discussed above, φ may be set $= \pi/2$. Further $g(\theta)$ may be set 0 if a positive σ is combined with a negative $\bar{\tau}_{c,z}$, see (2c). Taking into account the first restriction is marked by "φ" in Fig.2, the second restriction by "θ". The experimental data of Sattler et al. (10) for a Fe-30Ni steel are entered in Fig.2 and fit well to curve 1 with $\sigma_f = 750$ N/mm^2 . Curve 4 can be related with the famous "Greenwood/Johnson" relation

$$\varepsilon = \bar{\delta}/3 + 5/6\ .\ \sigma/\sigma_f\ .\ \bar{\delta} \qquad \text{(10a)}$$

for small values σ/σ_f . Comparison between the curves 1 and 4 of Fig.2 demonstrates the strong influence of γ . The ε-σ- relation in compression differs only slightly from that one in tension and is suppressed in Fig.2. It may be of interest that for small ratios σ/σ_f an analytical ε-σ- relation can be given with might be named the "extended" Greenwood/Johnson relation

$$\varepsilon = \delta/3 + 5/6\ .\ \sigma/\sigma_f\ .\ (\bar{\delta}^2 + 3/4\ \bar{\gamma}^2)^{1/2} \qquad \text{(10b)}$$

For details see (6), (7).

Specimen under pure shear, $\tau_{xz} = \tau$

$\bar{\xi}_N$ has the following components diffrent from 0:

$\bar{\varepsilon}_{N,x} = \varepsilon$, $\bar{\varepsilon}_{N,y} = x$, $\bar{\varepsilon}_{N,z} = \varepsilon$, $2\varepsilon + x = \delta$, $\bar{\varepsilon}_{N,xz} = \bar{\varepsilon}_{N,zx} = \Gamma/2$.

For $\bar{\delta} = 0.04$ and $\bar{\gamma} = 0.0$ rsp. 0.2 a relation between the relative global shear Γ/ε_{CV} ($\varepsilon_{CV} = 0.1318$) and τ / τ_f ($\tau_f = \sigma_f / \sqrt{3}$) is depicted in Fig.3. It is interesting that the axial strain components are influenced to a certain degree by the shear load. This can be seen by a certain deviation of x from $\delta / 3$, see the Γ/ε_{CV}-x-relation.

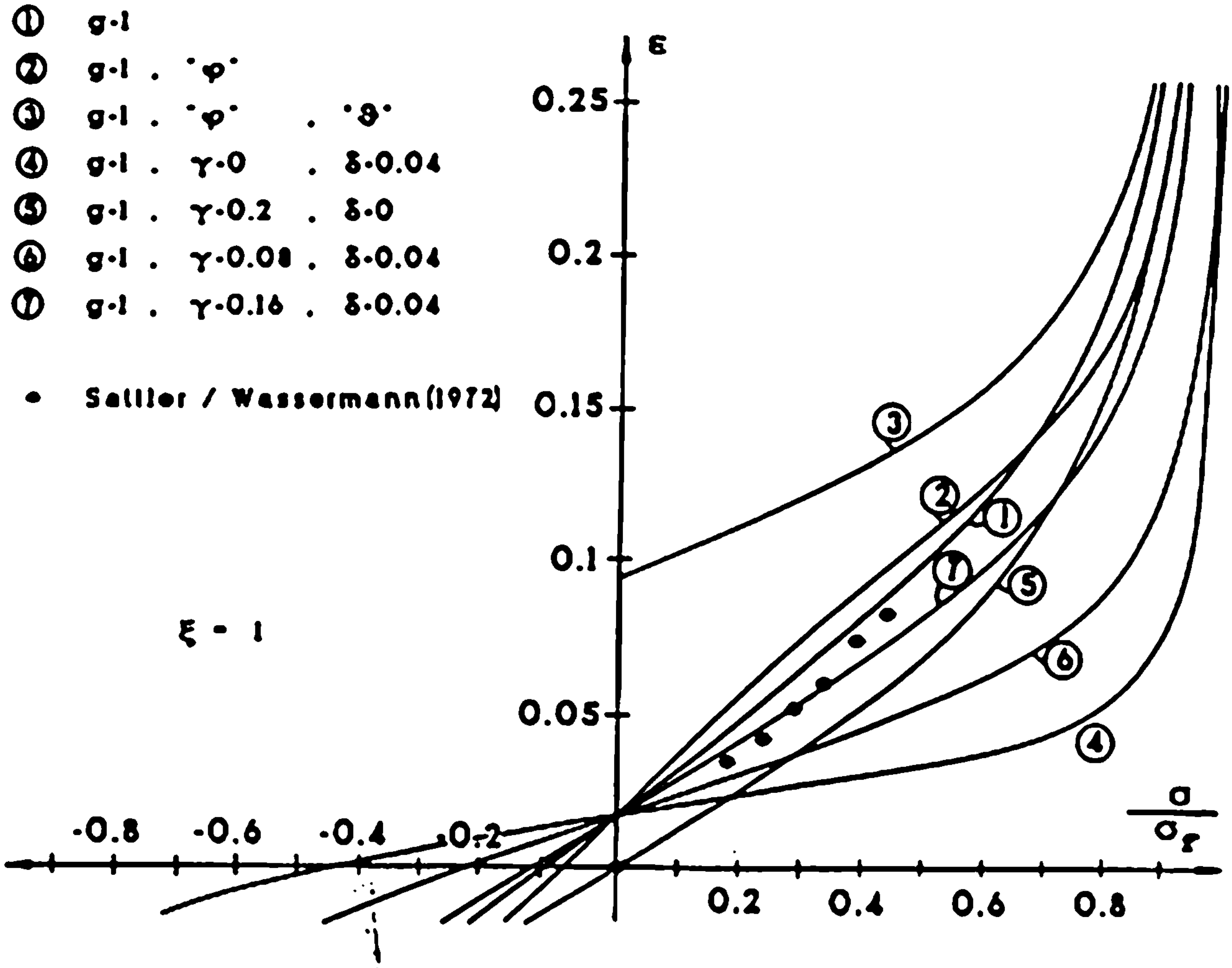
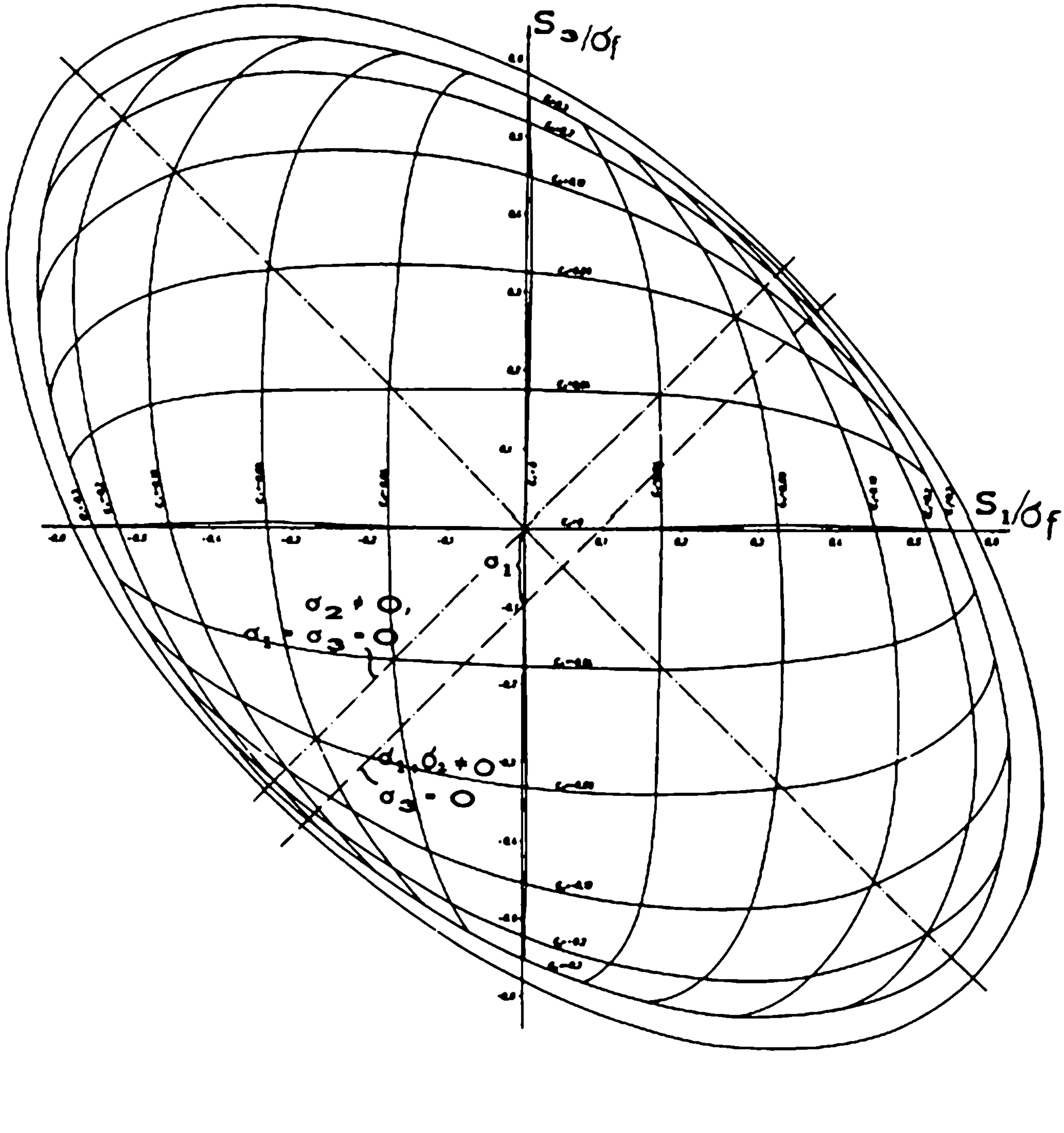


Fig.2: ε-δ-relation for a specimen under a unaxial stress state

Specimen under a threeaxial principal stress state $\sigma_x = \sigma_1$, $\sigma_y = \sigma_2$, $\sigma_z = \sigma_3$

Corresponding to the three principal stresses three principle strains ε_1 , ε_2 , ε_3 exist with $\varepsilon_1+\varepsilon_2+\varepsilon_3 = \bar{\delta}$. Since plastification is independent on p the analysis is performed with the deviator components S_1, S_3 ($S_2 = -(S_3 + S_1)$). Finally, in the case of a unaxial loading σ_2, $\sigma_1 = \sigma_3 = 0$, it follows $S_1 = S_3 = -S_2/2$. Lines of constant ε_1 and ε_3 are depicted in Fig.4. It can be seen that within the "square" $\varepsilon_1 = \pm 0.08$, $\varepsilon_3 = \pm 0.08$ only a small influence of σ_2 , σ_3 rsp. σ_1 , σ_2 exists. A linear, vectorial superposition of the strain components is possible.



<u>Fig.4</u>: Axial strains for a specimen under a threeaxial stress state

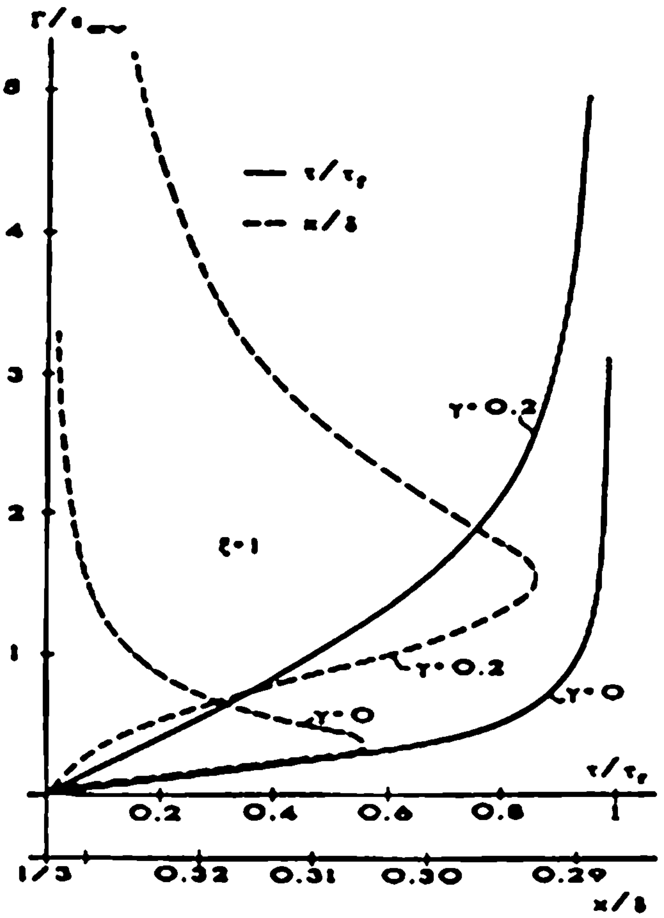


Fig.3: Γ-τ and Γ-x-relation for a specimen under pure shear

Conclusion

A semianalytical model for TRIP is presented taking into account both the plastic accommodation of the transformation strains and the selection of martensite variants. An extension to a threeaxial stress state is possible as a starting point for a "TRIP-law" formulation which can be used in numerical procedures such as the finite element method. The yield stress of the microregion, σ_f works as an "adaption parameter" in relation to experimental data.

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