CADSMA: Computer Aided Design of Shape Memory Alloys
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I. Introduction

An engineer faced with the design of a linear elastic spring or spring system can easily find published, easy-to-use mechanical properties for the materials of construction. Besides, the design formulas are simple and straight forward.

Designing with shape memory alloys is far more complicated than with non-transforming materials. Shape memory events take place in three dimensions (stress-strain-temperature). The material characteristics (stress-strain) are non-linear, temperature and even history dependent. So are the spring characteristics (force-displacement-temperature) of these materials.

2. Computer aided design of shape memory applications

The design formulas are very complex due to the non-linear, temperature and history dependent behaviour. Hence the design becomes difficult and time consuming. Computer aided (CAD) can be used here to simplify the procedures and to save a lot of manual computation time.

Several design programs have been developed:
1) helical (extension and compression) springs and spring systems (fig.1.);
2) coned-disk or Belleville springs (fig.2.) these springs consist essentially of circular disks dished to a conical shape;
3) leaf springs (fig.3.);
4) tube connectors (fig.4.).

The main lines of the working method are equivalent in all the design programs. This working method is illustrated by the easy-to-understand example of a helical spring acting against a bias load.

3. Helical shape memory spring acting against a bias load

The design problem is outlined in fig.5.. The shape memory spring acting against a constant load \( F \), should deflect by an amount \( \delta \) between the low temperature \( T_1 \) and the high temperature \( T_2 \). Hence the application input parameters are the load \( F \), the deflection \( \delta \) and the temperatures \( T_1 \) and \( T_2 \). As output the design should give the spring geometry and specify the shape memory alloy.

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Fig. 1. a) Helical spring
   b) Helical spring system

Fig. 2. Belleville spring

Fig. 3. Leaf springs
The geometry of a helical extension or compression spring is characterized by (fig.1.) :

1) spring diameter $D$ ;
2) wire diameter $d$ ;
3) number of coils $n$ ;
4) winding angle $\alpha$ .

From these can be determined :

5) spring index $c = D/d$ and
6) length of the spring wire $l = \pi n D$. 
The shape memory alloy specifications are:

1) the alloy number;
2) the two way shape memory strain;
3) the maximum allowed shear stress.

4. Solution to the design of a loaded, helical shape memory spring

The formulas used in the computer aided design (CAD) programs are based on the so-called classical theory. This theory is based on the assumption that an element of an axially loaded helical spring behaves essentially as a straight bar under pure torsion. The theory neglects effects due to bar curvature and direct shear loading. In that case, each element of the spring coil is subject to a torque \( F \times D / 2 \).

Two kinds of equations can be written down. The first kind relates the displacements of the application with the internal strains. The second kind gives the equilibrium between the external loads and the internal stresses. In case of a helical spring the relation between the displacement \( x_r \) at a given temperature \( T \) and the maximum shear strain \( \gamma_{st} \) (at the surface) is given by:

\[
x_r = (c \times x \times n \times D) \times \gamma_{st}
\]

or \( x_r = \mu \times \gamma_{st} \)

with \( \mu = c \times x \times n \times D \)

in which \( \mu \) is only dependent on the spring geometry. From this equation the deflection \( \delta \) between the temperatures \( T_1 \) and \( T_2 \) can be calculated:

\[
\delta = x_{r_2} - x_{r_1} = \mu \times (\gamma_{st_2} - \gamma_{st_1})
\]

The momentum equilibrium results in:

\[
F = - \left( \frac{d}{2} \right) \int_0^D x \pi r^2 x r \, dr
\]

In case of linear elastic material equ.1 and \( \delta \) give a direct, simple relationship between \( F \) and \( x_r \):

\[
F = (d/8nc^3) \times G \times x_r = k \times x_r
\]

in which \( k \) is the spring stiffness.

For non-linear materials equ.5 can be rewritten and divided into a spring geometry dependent and a material dependent part:

\[
F = \frac{x d^3}{8D} \times \left[ \frac{4}{(\xi_0)^3} \times \int_0^{\xi_2} \tau \, d\xi \right]
\]

or \( F = \int \tau \, d\xi \)
The integral in (10) is independent of the spring geometry. This means that
the modified shear stress $\tau_m (= f(T, g^*)$) is also a material characteristic
which can be calculated independent of the spring geometry.

5. Design steps

The design can now be subdivided into four steps. The first step is to
measure the shear stress–strain loading curves at various temperatures and for
different alloys. For this, wire torsion or spring tests can be used.
Secondly, the shear stress–strain curves are stored on hard or floppy disk (by
means of a general program GRNER). The third step is calculating (equ.10) the
modified shear stress–strain curves (TORS). By doing this in a separate step
a lot of time is saved during the last step. The first three steps are
independent of spring geometry and are the preparatory work before the fourth
step can be started. The fourth step is the heart of the matter: the
computer aided design (CADSMAR). By means of the equations 2, 3, 4, 8, 9 and 10
the computer solves the design problem.

6. CADSMAR

The alloy number, the low ($T_1$) and high ($T_2$) temperature, the two way memory,
and the maximum allowed shear stress are selected by the user in interaction
with the computer screen and keyboard. After this the computer uses equ.10 to
calculate $g_1$ and $g_2$. The next input is the bias load $F$ and the deflection $\delta$.
With equ.2, 3, 4, 8, 9 and 10 the computer calculates the spring diameter $D$
and the length of the spring wire as a function of the wire diameter. This
"design chart" (fig.6) is presented on the screen. The last step is the
selection of the appropriate wire diameter. The user selects the optimum wire
diameter and the information about the spring is presented on the screen.

![CADSMAR design chart](image-url)
7. Advantages

The basic idea is that these programs should be user-friendly. This means that the programs are now easy-to-use and self-explanatory. Nevertheless the CADSMA is only a fast and user-friendly aid to the design of shape memory applications. The computer aided design is used to solve the complicated and time consuming computations. The intelligent work (selection of the shape memory alloy, wire diameter, ...) has still to be done by the design engineer although also this occurs in interaction with the computer screen. Besides, the computer presents the graphs and information clearly on the screen and eventually on the printer.

8. Deviations between actual and calculated behaviour

Although the programs are user-friendly, a shape memory specialist is necessary to know if the program is suitable for the customers problem because of possible deviations between actual and calculated behaviour. Possible deviations are due to the different thermomechanical way followed during the calculation and in reality, the instabilities of the shape memory effect and hysteresis effects.

In case deviations due to the different thermomechanical way should occur, they are expected at temperatures close to the transformation temperatures and at larger shape memory or pseudoelastic deformations. Those δ-deviations can be adjusted after experimental control. This is easily done by adjusting the amount of coils n because of the linear relationship between δ and n (equ.4).

Some possible instabilities of the shape memory effect are transformation creep, degradation and ageing of the memory effect.

Thirdly, use of the loading curves can only result in a primary design of a shape memory spring. It gives no information on the hysteresis of the designed spring.

10. Conclusion

In its simple form the CADSMA-programs offer a user-friendly and fast way to design the preliminary shape of any shape memory device. Dependent on the complexity or required accuracy of the dimensions of the shape memory element and its thermomechanical response function the computer designed shape may have to be corrected in a simple way as in the case of a helical spring or on an iterative "trial-and-error"-basis. In the latter case the experience of the shape memory specialist plays a key-role.

The main problems that limit presently the calculation of the definite shape in one step are the history dependent material characteristics, the hysteresis between the reverse and the forward transformation and the instabilities of the shape memory effect.

Nevertheless, with increasing experience in SME-design more sophisticated CADSMA-programs can be developed, taking into account the experimental evidences. As a consequence, the design of CADSMA-programs will develop in the direction of expert-systems.

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